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READINESS AND THE OPTIMAL REDEPLOYMENT OF RESOURCES

Seymour Kaplan

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13. ABSTRACT

This paper considers the problem of the optimal redeployment of a resource among different geographical locations. Initially, it is assumed that at each location, i, $i = 1, ---n$, the level of availability of the resource is given by $a_i \geq 0$. At time $t > 0$, requirements $R_i(t) \geq 0$ are imposed on each location which in general will differ from the a_i. The resource can be transported from any one location to any other in magnitudes which will depend on t and the distance between these locations. It is assumed that $\Sigma R_i > \Sigma a_i$.

The objective function considers, in addition to transportation costs incurred by reallocation, the degree to which the resource availabilities after redeployment differ from the requirements. We shall associate the unavailabilities at the locations with the unreadiness of the system and discuss the optimal redeployment in terms of the minimization of the following functional forms:

$\sum_{j=1}^{n} k_j(R_j-y_j)$ + transportation costs, $\operatorname{Max}_j \left[k_j(R_j-y_j)\right]$ + transportation costs, and

$\sum_{j=1}^{n} k_j(R_j-y_j)^2$ + transportation costs. The variables y_j represent the final amount of the resource available at location j. No benefits are assumed to accrue at any location if $y_j > R_j$. A numerical three location example is given and solved for the linear objective.

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Problem

Suppose there are n geographical locations where an organization requires varying levels of a resource (manpower, fuel, equipment). The requirements for this resource are assumed to change as sudden demands for the resource brought about by changing economic, political, or natural conditions are created. For example, natural disasters such as floods may create a need for certain types of rescue equipment at various flood locations. To satisfy the needs at any one location, the resource may be obtained locally or from any other locations where availability exists. There are limitations on the magnitudes of the resource which may be transported from location i to location j. These limitations depend on the allowable time t for reallocation to take place as well as the distance between locations. In the present problem, t is fixed and given so that the limitations are given constants.

We shall consider several types of objective functions (to be discussed below) which we wish to partially associate with the degree of unreadiness of the system. That is, we consider several different measures of unreadiness and investigate how the optimal reallocation changes with these measures. In addition to the costs incurred as a result of unreadiness, we assume that the physical process of reallocation also results in transportation costs. The weighted sum of these two types of costs will constitute the objective function. In each case, it is assumed that ending up with more of a resource than required at a location does not result in any benefits. Also the problem is deterministic and contains no stochastic elements.

Definitions

Let

x_{ij} = the amount of the resource to be transported from location i to location j

y_j = the final level of the resource at location j

c_{ij} = the cost of transporting one unit of the resource from location i to location j ; $c_{ij} \geq 0$.

a_i = the initial availability of the resource at location i; $a_i \geq 0$.

$R_j(t)$ = the requirement of the resource by time t at location j ; $R_j(t) = R_j \geq 0$, where t is assumed fixed.

$M_{ij}(t)$ = the maximum allowable magnitude of the resource that can be shipped from i to j in an interval of length t . $M_{ij}(t) = M_{ij} \geq 0$.

k_j = the relative importance of location j insofar as resource insufficiency at that location is concerned. The greater k_j , the more critical an insufficiency at location j ; $k_j \geq 0$.

It is assumed that $$\sum_{j=1}^{n} R_j \geq \sum_{i=1}^{n} a_i \quad .$$

Problem Formulation

The problem to be solved can be set up in a transportation type format where each location is considered as both an origin and destination. The constraints state that the amount of product to be sent from location

i cannot exceed a_i, the amount received by any location is equal to y_j, where y_j cannot exceed R_j and the amount shipped from any location to any other is limited by the M_{ij}. Thus, we obtain:

$$(1) \quad \text{Min } z = f(R_j, y_j) + \sum_i \sum_j c_{ij} x_{ij}$$

$$\sum_{j=1}^{n} x_{ij} \leq a_i \; ; \; i = 1 \ldots n$$

$$\sum_{i=1}^{n} x_{ij} = y_j \; ; \; j = 1, \ldots n$$

$$y_j \leq R_j \quad j = 1 \ldots n$$

$$x_{ij} \leq M_{ij} \quad \text{all } i, j$$

$$x_{ij} \geq 0 \text{, all } i, j \; ; \; y_j \geq 0 \; j = 1, \ldots n.$$

The objective will be referred to as the unreadiness function and we shall consider and discuss several different mathematical forms of this function. Note that the y_j are problem variables. If we take a linear objective function of the form

$z = \sum_{j=1}^{n} k_j (R_j - y_j) + \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$ it will be seen that the problem can be reduced to a standard capacitated transportation problem.

Let $u_j = R_j - y_j$. Then (1) becomes:

$$(2) \quad \text{Min } z = \sum_{j=1}^{n} k_j u_j + \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$$

$$\sum_{j=1}^{n} x_{ij} \leq a_i \quad i = 1, \ldots n.$$

$$\sum_{i=1}^{n} x_{ij} + u_j = R_j \quad j = 1, \ldots n$$

$$x_{ij} \leq M_{ij}$$

$$x_{ij} \geq 0 , \; u_j \geq 0$$

If the u_j are considered as the amounts shipped from an additional fictitious origin then the problem can be considered as one where the unreadiness costs (the k_j) are associated with shipping from the additional origin. If the availability at this origin is considered to be a_{n+1}, where a_{n+1} may be set equal to some large value ($\sum_{j=1}^{n} R_j$ will do) , then an additional origin constraint of the form

$$\sum_{j=1}^{n} u_j \leq a_{n+1}$$

puts the problem into a format with $n + 1$ origin constraints and n destination constraints. The problem may be interpreted insofar as unreadiness is concerned, as one where we wish to avoid shipping from origin (n+1) as much as possible. If the $\sum_{i=1}^{n} x_{ij} = R_j$ then the requirement at j can be met without unreadiness penalty. If $\sum_{i=1}^{n} x_{ij} < R_j$, then a penalty due to unreadiness is incurred at location j. Or, one may state the problem as one where unreadiness costs are only associated with slack variables in the destination constraints when the problem is cast in the form:

$$(3) \quad \text{Min} \sum_{i=1}^{n+1} \sum_{j=1}^{n} c_{ij} x_{ij}$$

subject to:

$$\sum_{j=1}^{n} x_{ij} \le a_i \ , \ i = 1, \ldots n + 1 \ .$$

$$\sum_{i=1}^{n+1} x_{ij} \le R_j \ , \ j = 1, \ldots n \ .$$

$$x_{ij} \le M_{ij}$$

$$x_{ij} \ge 0 \quad i = 1, \ldots n + 1 \ ; \ j = 1, \ldots n \ .$$

and where $c_{n+1,j} = k_j$.

To finally state the problem in the standard transportation format, consider an additional fictitious destination such that the slack variables of the origin constraints represent the amounts of the resource shipped to this destination. Call the slack variables $x_{i,n+1}$ where $i = 1, \ldots n+1$. Then the problem becomes

$$\text{Min} \sum_{i=1}^{n+1} \sum_{j=1}^{n+1} c_{ij} x_{ij}$$

subject to:

$$\sum_{j=1}^{n+1} x_{ij} = a_i \ , \ i = 1, 2, \ldots n+1$$

$$\sum_{i=1}^{n+1} x_{ij} = R_j \ , \ j = 1,2,\ldots n+1$$

$$0 \le x_{ij} \le M_{ij} \quad \text{all } i,j$$

In this problem, $a_{n+1} = \sum_{j=1}^{n} R_j$

$$R_{n+1} = \sum_{i=1}^{n+1} a_i - \sum_{j=1}^{n} R_j = \sum_{i=1}^{n} a_i \text{ (so that } \sum_{i=1}^{n+1} a_i = \sum_{j=1}^{n+1} R_j) .$$

Also the $M_{jj} = \text{Min }(a_j,R_j)$ so that if $R_j < a_j$ location j will only end up with R_j , whereas if $R_j \ge a_j$, the entire availability can remain. Since the x_{ii} represent shipping from a location to itself, we shall assume that $c_{ii} = 0$. Also, we take $M_{n+1,j} = R_j$ so that if necessary, up to R_j units will be sent to destination j from origin $n+1$; and $M_{i,n+1} = a_i$. Finally $C_{i,n+1} = 0, \ i = 1, \ldots n+1$.

Assuming that a feasible solution exists, the above problem can be solved as a capacitated transportation problem with $n+1$ origins and $n+1$ destinations.

When the objective is in the form $z = \underset{j}{\text{Max}} [k_j(R_j-y_j)] + \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}x_{ij}$,

we can convert the problem to a linear program, but not a transportation problem by noting that

$$z = \underset{j}{\text{Max}} \left[k_j(R_j-y_j) + \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} \, x_{ij} \right]$$

After making the transformation $u_j = R_j - y_j$ as before, the problem is equivalent to the following linear program:

(3) Min v

$$k_j u_j + \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} \leq v \ , \ j = 1, 2, \ldots n$$

+ the other constraints of (2) .

The above objective is often referred to as a minimax objective and can occur in curve fitting and regression problems as well as in the present context. See [4] for example.

With a quadratic objective of the form $z = \sum_{j=1}^{n} k_j (R_j - y_j)^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$, the problem may be solved as a quadratic program after letting $u_j = R_j - y_j$, since the quadratic form $\sum_{j=1}^{n} k_j u_j^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$ is positive definite (k_j, $c_{ij} \geq 0$ and the form cannot have the value zero since $a_j < R_j$) . Wolfe's method for quadratic programming is a convenient procedure to use [4] .

It should be noted that with the min-max objective and the quadratic, the problem can be solved via simplex tableaus. The min-max problem requires n additional constraints above the n+1 origin and n+1 destination constraints where n = the number of locations. The quadratic problem, via the Wolfe technique requires $(n+1)^2$ additional constraints, corresponding to the number of variables in the problem with n+1 origins and n+1 destinations.

Objective Function

The objective function is one which transforms the cost of unreadiness into costs associated with transportation and assumes such a cost is additive to the transportation costs. The great difficulty of such a procedure is of course in developing meaningful empirical procedures for such a transformation. If we consider that the objective functions represent a disutility to the organization then we are assuming that the disutility due to unreadiness is additive to that of transportation cost. We are here essentially dealing with the problem of decision making with respect to multiple objectives and encounter the usual difficulties when doing so. See [1] for examples.

In the context of the present problem, we consider the disutility due to unreadiness to be the major concern and include the transportation costs because the formulation is more general, no difficulties are added to the problem in solution, and because such costs may in fact influence the optimal reallocation if some of them are sufficiently large. However, the problem can also be considered with all $c_{ij} = 0$ so that the unreadiness disutility is the only consideration.

The linear objective function for unreadiness assumes that the overall unreadiness is measured as a weighted sum of the insufficiencies in the supply of the resource, the weight taken over the different geographical locations. The weights may be normalized and could be estimated by a variety of techniques relating to the problem of decision making with respect to multiple criteria. In essence, we are assuming that the organization has an additive linear disutility function with respect to resource insufficiencies.

With the objective function which minimizes the maximum insufficiency, the measure of unreadiness is related to the worst possible insufficiency and is essentially a "conservative" criterion. For any optimum solution to this problem, the average insufficiency taken over locations will in general be expected to be greater than with the previous criterion.

With the quadratic unreadiness objective, the measure of course penalizes locations more severely for insufficiencies > 1 than does the linear function. Here again the assumption is of an additive utility function taken over locations.

Much of which type of objective, of the three discussed, as well as others, will of course depend on the nature of the resource and how it is combined or used with other resources. Resources such as aircraft fuel may, in short supply, penalize short run operations much more severely than resources such as certain food items. In the latter case the min-max objective might be more appropriate since we might be interested in the shortage of such resources not getting out of "control" anywhere and trying to keep the worst possible shortage as low as possible.

Extension to Multiple Resources

If we assume that a simultaneous shortage of two or more resources affect the ability of the organization to carry out its mission to an extent greater than or equal to that of one resource, then we can postulate a variety of models for describing this simultaneous shortage.

Much will depend on how the resources interact with each other in carrying out functions. Thus, certain levels of pilots and airplane shortages simultaneously may not affect the readiness much more than the given shortage level of just one of these whereas corresponding shortages of pilots and ASW equipment may affect the readiness of a unit in an additive manner.

An additive situation would seem appropriate when the resources in question were used for what may be termed "independent" missions where the resources needed for one mission are unrelated to those needed for the others. Of course in a real sense no two missions of an organization during a particular period of time are truly independent. However, if the additive model seems appropriate, the problem could be handled by including another summation in the objective function over resources and adding additional constraints for each resource. Thus, the form of the objective function for the linear unreadiness model would be:

$$\text{Min } z = \sum_{\ell=1}^{q} \sum_{j=1}^{n} k_{\ell j} (R_{\ell j} - y_{\ell j}) + \sum_{\ell=1}^{q} \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij\ell} x_{ij\ell}$$

where there are q resources, and where the subscript ℓ refers to the ℓ th resource.

Non-additive situations would involve certain non linearities in formulation and are beyond the scope of this paper.

<u>Example</u>

We shall illustrate the solution for the linear objective function with an example. Consider the following reallocation problem with three locations, set up in a tableau format as follows:

Location	1	2	3	a_i
1	0 4	0.01 2	0.02 2	4
2	0.02 3	0 6	0.02 3	6
3	0.02 1	0.01 1	0 7	7
R_j	6	8	8	$\Sigma a = 17$
k_j	0.4	0.3	0.2	$\Sigma R_j = 22$

The numbers in the upper left of each cell of the 3 x 3 location matrix indicate the transportation costs while those in the lower right indicate the capacity of each route i.e. $c_{12} = 0.01$, $x_{12} \leq 2$. The overall requirement is for 22 units whereas the overall availability is 17.

We shall solve the problem using the primal-dual method for the capacitated transportation problem and the notation and tableau format of Hadley [2].

The problem requires 6 tableaus for solution. They are shown in the Appendix. The optimal minimum cost solution is found by transporting one unit from location three to location one and one unit from location three to location two. The optimal redeployment can be read off the final tableau reproduced below. The values in the circles of the fourth row cells (04) corresponding to the fictitious origin, show the final deficiencies at each location ie. $R_1 - y_1 = 1$, $R_2 - y_2 = 1$, $R_3 - y_3 = 3$ (The 17 is the excess going to the fictitious destination). The values in the circles on the off-diagonal elements indicate the redeployments. In this problem the value of the objective function is $z_{min} = 1.33$.

	$u_i \backslash v_j$	D_1		D_2		D_3		D_4		a_i	x_s^1	δ_i	γ_i
		0.00		0.00		-0.20		-0.40					
O_1	0.00	0.00	4	0.01	2	0.02	2	0.00	4	4			
		4									0		
O_2	0.00	0.02	3	0.00	6	0.02	3	0.00	6	6			
				6							0		
O_3	0.20	0.02	1	0.01	1	0.00	7	.00	7	7			
		1	-0.18	1	-0.19	5					0		
O_4	0.40	0.40	6	0.30	8	0.20	8	0.00	22	22			
		1		1	-0.10	3		17			0		
	b_j	6		8		8		17					
			0		0		0		0	0			
	σ_j												
	ρ_j												

Table 1: Final tableau for the Example.

For notation, see p. 358 and p. 397 of [2]

Solution $x_{31} = 1$, $x_{32} = 1$, $z = 1.33$

References

[1] Fishburn, Peter, Decision and Value Theory, John Wiley and Sons, Inc., 1964.

[2] Hadley, G., Linear Programming, Addison-Wesley Publishing Company, Inc., 1962.

[3] ________, Nonlinear and Dynamic Programming, Addison-Wesley Publishing Company, Inc., 1964.

[4] Wagner, Harvey M., Principles of Operations Research, Prentice-Hall, Inc., 1969.

APPENDIX

Tableau 1

	$u_i \backslash v_j$	D_1		D_2		D_3		D_4		a_i	x_s^i	δ_i	γ_i
		0.0		0.0		0.0		0.0					
O_1	0.0	0.00	4	0.01	2	0.02	2	0.00	4	4	0		
		4											
O_2	0.0	0.02	3	0.00	6	0.02	3	0.00	6	6	0		
				6									
O_3	0.0	0.02	1	0.01	1	0.00	7	0.00	7	7	0		
						7							
O_4	0.0	0.40	6	0.30	8	0.20	8	0.00	22	22	5	5	4
									17				
	R_j	6		8		8		17					
	X_{sj}	2		2		1		0		5			
	e_j							5					
	p_j							4					

$h = 0.20$

Tableau 2

	u_i \ v_j	D_1	D_2	D_3	D_4	a_i	x_s^i	δ_i	ν_i
		0.0	0.0	0.0	-0.20				
O_1	0.0	0.0 4 4	0.01 2	0.02 2	0.00 4	4	0		
O_2	0.0	0.02 3	0.00 6 6	0.02 3	0.00 6	6	0		
O_3	0.0	0.02 1	0.01 1	0.00 7 7	0.00 7	7	0	4	3
O_4	0.20	0.40 6	0.30 8	0.20 8 1	0.00 22 17	22	4	4	4
	R_j	6	8	8	17				
	x_{sj}	2	2	0	0		4		
	e_j			4	4				
	p_j			4	4				

$h = 0.01$

Tableau 3

	u_i \ v_j	D_1: 0.0	D_2: 0.0	D_3: -0.01	D_4: -0.21	a_i / x_s^i	δ_i	γ_i
O_1	0.0	0.0 4 4	0.01 2	0.02 2	0.00 4	4 0		
O_2	0.0	0.02 3	0.00 6 6	0.02 3	0.00 6	6 0		
O_3	0.01	0.02 1	0.01 1 1 -0.01	0.00 7 6	0.00 7	7 0	1	3
O_4	0.21	0.40 6	0.30 8	0.20 8 2	0.00 22 17	22 3	2	4
R_j / x_{sj}		6 2	8 1	8 0	17 0	3		
a_j				3	3			
o_j				4	4			

$h = 0.01$

Tableau 4

	u_i \ v_j	D_1		D_2		D_3		D_4		a_i	x_s^i	δ_i	γ_i
		0.00		0.0		-0.02		-0.22					
O_1	0.00	0.0	4	0.01	2	0.02	2	0.00	4	4			
		4									0		
O_2	0.00	0.02	3	0.00	6	0.02	3	0.00	6	6			
				6							0		
O_3	0.02	0.02	1	0.01	1	0.00	7	0.00	7	7			
		1	-0.08	1	-0.09	5					0	1	3
O_4	0.22	0.40	6	0.30	8	0.20	8	0.00	22	22			
						3		17			2	1	4
	R_j	6		8		8		17					
	x_{sj}		1		1		0		0		2		
	s_j						1		1				
	p_j						4		4				

$h = 0.08$

Tableau 5

	u_i \ v_j	D_1		D_2		D_3		D_4		a_i	x_s^i	δ_i	γ_i
		0.00		-0.10		-0.10		-0.30					
O_1	0.00	0.C	4	0.01	2	0.02	2	0.00	4	4			
		4									0		
O_2	0.00	0.02	3	0.00	6	0.02	3	0.00	6	6			
				6							0		
O_3	0.10	0.02	1	0.01	1	0.00	7	0.00	7	7			
		1	-0.18	1	-0.19	5					0	1	2
O_4	0.30	0.40	6	0.30	8	0.20	8	0.00	22	22			
				1	-0.10	3		17			1	1	4
	R_j	6		8		8		17					
	x_{sj}				0		0		0		1		
	e_j				1		1		1				
	ρ_j				4		4		4				

$\lambda = 0.10$

Tableau 6

	$u_i \backslash v_j$	D_1		D_2		D_3		D_4		a_i	x_s^i	δ_i	γ_i
		0.00		0.00		-0.20		-0.40					
O_1	0.00	0.0	4	0.01	2	0.02	2	0.00	4	4			
		4									0		
O_2	0.00	0.02	3	0.00	6	0.02	0	0.00	6	6			
				6							0		
O_3	0.20	0.02	1	0.01	1	0.00	7	0.00	7	7			
		1		1		5							
O_4	0.40	0.40	6	0.30	8	0.20	8	0.00	22	22			
		1		1		3		17			0		
	R_j	6		8		8		17					
	x_{sj}		0		0		0		0		0		
	ϵ_j												
	ρ_j												